Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

$$t^6-\frac{2b^2}{a^2}t^4+\frac{2b^2}{a^2}t^3-\frac{2b^2}{a^2}t^2+\frac{b^4}{a^4}=0.$$

Solving this for numerical values of a and b, we get the values of x and y from the above expressions.

The same equation may be arrived at by putting $x=r\cos\theta$, $y=r\sin\theta$. The given equation then changes into $a^2\cos\theta=2r^2\cos^2\theta-a^2$; $b^2\sin\theta=2r^2\sin^2\theta-b^2$. Adding, we get $a^2\cos\theta+b^2\sin\theta=2r^2-(a^2+b^2)$, whence $r^2=\frac{1}{2}[a^2\cos\theta+b^2\sin\theta+a^2+b^2]$. Also, $r^2=\frac{1}{2}.\frac{a^2\cos\theta+a^2}{\cos^2\theta}$.

Equalizing, changing into the tangent function, the latter being denoted by t, we obtain the same equation as above.

IV. **Solution by H. C. WILKES, Skull Run, West Virginia.**

Putting $x^2+y^2=s^2$; then from (1), $a^2(x+s)=2sx^2$, and from (2), $b^2(y+s)=2sy^2$. Any rational value for s will give integral [?] fractional values for a^2 and b^2. Let $s=5$, $a^2=45/4$, and $b^2=160/9$; $s=13$, $a^2=325/9$, and $b^2=3744/25$; $s=17$, $a^2=2176/25$, and $b^2=3825/16$.

67. **Proposed by COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tennessee.**

Prove that $\cos\frac{n\pi}{7}+\cos\frac{3n\pi}{7}+\cos\frac{5n\pi}{7}=\frac{1}{2}$ or $-\frac{1}{2}$, according as n is *odd* or *even*, [and not a multiple of 7].

I. **Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas-Texas.**

$$\sin\frac{2n\pi}{7}=2\sin\frac{n\pi}{7}\cos\frac{n\pi}{7}.$$

$$\sin\frac{4n\pi}{7}-\sin\frac{2n\pi}{7}=2\sin\frac{n\pi}{7}\cos\frac{3n\pi}{7}.$$

$$\sin\frac{6n\pi}{7}-\sin\frac{4n\pi}{7}=2\sin\frac{n\pi}{7}\cos\frac{5n\pi}{7}.$$

$$\therefore\cos\frac{n\pi}{7}+\cos\frac{3n\pi}{7}+\cos\frac{5n\pi}{7}=\frac{\sin\frac{6}{7}n\pi}{2\sin\frac{1}{7}n\pi}=\frac{\sin(n\pi-\frac{1}{7}n\pi)}{2\sin\frac{1}{7}n\pi}$$

$=-\frac{1}{2}\cos n\pi=\pm\frac{1}{2}$, according as n is odd or even.

II. **Solution by J. SCHEFFER, A. M., Hagerstown, Maryland.**

Employing the well-known formula

$$\sum_{n=1}^{n=n}\cos[a+(n-1)b]=\frac{\cos[a+\frac{1}{2}(n-1)b]\sin\frac{1}{2}nb}{\sin\frac{1}{2}b},$$

and putting $b=2a$, $n=3$, we have $\cos a+\cos 3a+\cos 5a=\frac{\cos 3a\sin 3a}{\sin a}=\frac{1}{2}\frac{\sin 6a}{\sin a}$.

But either $\frac{1}{2}\frac{\sin 6a}{\sin a}=\frac{1}{2}\frac{\sin 6a+\sin a}{\sin a}-\frac{1}{2}=\frac{\sin\frac{7}{2}a.\cos\frac{5}{2}a}{\sin a}-\frac{1}{2}$,

$$\text{or, } \frac{1}{2}\frac{\sin 6a}{\sin a}=\frac{1}{2}\frac{\sin 6a-\sin a}{\sin a}+\frac{1}{2}=\frac{\cos\frac{7}{2}a.\sin\frac{5}{2}a}{\sin a}+\frac{1}{2}.$$

Putting $a=\frac{1}{7}n\pi$, we get in the former case $\frac{\sin\frac{1}{2}n\pi\cos\frac{5}{14}n\pi}{\sin\frac{1}{7}n\pi}-\frac{1}{2}$, and in the latter $\frac{\cos\frac{1}{2}n\pi\sin\frac{5}{14}n\pi}{\sin\frac{1}{7}n\pi}+\frac{1}{2}$. If n is *even*, $\sin\frac{1}{2}n\pi=0$, if *odd*, $\cos\frac{1}{2}n\pi=0$.

Q. E. D.

III. Solution by OTTO O. CLAYTON, A. B., Fowler, Indiana.

Unite 1st and 3rd terms of the left member ; then by factoring, we have,

$$(2\cos\tfrac{2}{7}n\pi+1)\cos\tfrac{3}{7}n\pi=\tfrac{1}{2}\text{ or }-\tfrac{1}{2}.$$

Substituting for $(2\cos\frac{2}{7}n\pi+1)$, we have $\frac{\sin\frac{3}{7}n\pi\cos\frac{3}{7}n\pi}{\sin\frac{1}{7}n\pi}=\frac{1}{2}$ or $-\frac{1}{2}$, from which $\frac{1}{2}\frac{\sin\frac{6}{7}n\pi}{\sin\frac{1}{7}n\pi}=\frac{1}{2}.\frac{\sin-\frac{1}{7}n\pi}{\sin\frac{1}{7}n\pi}=\frac{1}{2}$ or $-\frac{1}{2}$.

This being an identical equation the problem is proved ; for ratio

$$\frac{\sin-\frac{1}{7}n\pi}{\sin\frac{1}{7}n\pi}=1\text{ or }-1,\text{ according as } n \text{ is odd or even.}$$

IV. Solution by JOHN B. FAUGHT, A. M., Instructor in Mathematics in Indiana University, Bloomington. Indiana.

The equation (1), $(\cos\theta+i\sin\theta)^7=-1$, *i. e.*, $\cos 7\theta+i\sin 7\theta=-1$, is clearly satisfied when θ has either of the following values : $\frac{1}{7}\pi$, $\frac{3}{7}\pi$, $\frac{5}{7}\pi$, $\frac{7}{7}\pi$, $\frac{9}{7}\pi$, $\frac{11}{7}\pi$ and $\frac{13}{7}\pi$.

$\therefore$ (2), $(\cos n\theta+i\sin n\theta)^7=(-1)^n$ is satisfied by $\theta=\frac{1}{7}\pi$, $\frac{3}{7}\pi$, $\frac{5}{7}\pi$, $\frac{7}{7}\pi$, $\frac{9}{7}\pi$, $\frac{11}{7}\pi$ or $\frac{13}{7}\pi$, or $n\theta=\frac{1}{7}n\pi$, $\frac{3}{7}n\pi$, $\frac{5}{7}n\pi$, $\frac{7}{7}n\pi$, $\frac{9}{7}n\pi$, $\frac{11}{7}n\pi$ or $\frac{13}{7}n\pi$.

But (3), $(\cos n\theta+i\sin n\theta)^7=\cos^7 n\theta+7i\cos^6 n\theta\sin n\theta-21\cos^5 n\theta\sin^2 n\theta-35i\cos^4 n\theta\sin^3 n\theta+35\cos^3 n\theta\sin^4 n\theta+21i\cos^2 n\theta\sin^5 n\theta-7\cos n\theta\sin^6 n\theta-i\sin^7 n\theta=(-1)^n$.

$\therefore$ (4), $\cos^7 n\theta-21\cos^5 n\theta\sin^2 n\theta+35\cos^3 n\theta\sin^4 n\theta-7\cos n\theta\sin^6 n\theta=(-1)^n$.

Or (5), $64\cos^7 n\theta-112\cos^5 n\theta+56\cos^3 n\theta-7\cos n\theta-(-1)^n=0$, of which

$\cos\frac{1}{7}n\pi$, $\cos\frac{3}{7}n\pi$, $\cos\frac{5}{7}n\pi$, $\cos\frac{7}{7}n\pi$, $\cos\frac{9}{7}n\pi$, $\cos\frac{11}{7}n\pi$ and $\cos\frac{13}{7}n\pi$ are the ratio.

Now $\cos\frac{7}{7}n\pi = \mp 1$, according as n is *odd* or *even*, and $\cos\frac{13}{7}n\pi = \cos\frac{1}{7}n\pi$; $\cos\frac{9}{7}n\pi = \cos\frac{5}{7}n\pi$.

Hence we have (6), $(\cos n\theta \pm 1)(64\cos^6 n\theta \mp 64\cos^5 n\theta - 48\cos^4 n\theta \pm 48\cos^3 n\theta + 8\cos^2 n\theta \mp \cos n\theta + 1) = 0$, according as n is *odd* or *even*.

$\therefore$ (7), $2(\cos\frac{1}{7}n\pi + \cos\frac{3}{7}n\pi + \cos\frac{5}{7}n\pi) = \pm\frac{64}{64} = \pm 1$. Or $\cos\frac{1}{7}n\pi + \cos\frac{3}{7}n\pi + \cos\frac{5}{7}n\pi = \pm\frac{1}{2}$, according as n is *odd* or *even*.

We might deduce a number of equally interesting results, thus,

$$(\cos\tfrac{1}{7}n\pi . \cos\tfrac{3}{7}n\pi . \cos\tfrac{5}{7}n\pi)^2 = \tfrac{1}{64}.$$

$\therefore \cos\frac{1}{7}n\pi . \cos\frac{3}{7}n\pi . \cos\frac{5}{7}n\pi = \pm\frac{1}{8}$, when n is either *odd* or *even*, etc.

GEOMETRY.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

63. Proposed by ALFRED HUME, C. E., D. Sc., Professor of Mathematics, University of Mississippi, P. O., University of Mississippi.

A rectangular hyperbola cannot be cut from a right circular cone if the angle at its vertex is less than a right angle.

II. Solution by F. M. McGAW, A. M., Professor of Mathematics, Bordentown Military Institute, Bordentown, New Jersey.

Assume axes of coördinates at right angles.

(*a*) The equation of the surface of a cone with axis of z as axis of cone, and origin at the vertex of cone is

$$x^2 + y^2 - z^2\tan^2\tfrac{1}{2}v = 0 \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where v=angle at vertex.

(*b*) The equation of a plane to same axes and origin as above, in terms of its direction cosines and perpendicular from origin is

$$lx + my + nz = \phi \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2).$$

Eliminate z between (1) and (2), and then we have the conic